only negative values for any z_j. Then the derivative (2.5) is a positive-definite function in the variables $u, \xi_1, \ldots, \xi_{m-1}$ and it is evident that the unperturbed motion is asymptotically stable with respect to the variables $u, \xi_1, \ldots, \xi_{m-1}$ when α is odd, and unstable when α is even [5].

Thus, we obtain the following theorem.

Theorem 4. Let the equations of perturbed motion used in investigating the stability with respect to a part of the variables be reduced in the critical case of a single zero root to the form (2.3). Let also the Conditions A, B (1.4) and (2.4) all hold. Then, if $g(z_1, \ldots, z_p)$ assumes only the negative values, the unperturbed motion of the system (2.3) is asymptotically y-stable if α is odd, and y-unstable if α is even; if $g(z_1, \ldots, z_p)$ assumes only the positive values, then the unperturbed motion of the system (2.3) is y-unstable.

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ON THE STABILITY OF A LAMINAR FLOW OF A CONDUCTING FLUID FILM IN A TRANSVERSE ELECTRIC FIELD

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Averaged equations for defining in long-wave approximation the flow of a conducting fluid film on the vertical wall of a plane channel in a transverse electric field are derived. The convective and absolute instability of laminar flow is investigated with the use of these equations. It is established that periodic perturbations are intensified downstream of the flow only if their frequency does not exceed the critical frequency which depends on the electrohydrodynamic interaction parameter and the Weber number. It is shown that the electric field has a destabilizing effect owing to the increase of surface charge density in the vicinity of wave crests. This results in an increase of surface forces produced by the electric field at wave crests thereby reducing the stabilizing effect of surface tension forces. Proof is given of the absolute stability of the laminar flow of a

film in an electric field. The derived criterion of convective instability coincides qualitatively with experimental data obtained in investigations of film condensation of steam in vertical condensing systems in the presence of a transverse electric field.

Experimental investigations [1] of film condensation of steam in a vertical condenser with a transverse electric field had shown that when the intensity of the latter exceeds some critical value, the intensity of heat exchange considerably increases. This is explained by the electrohydrodynamic instability of flow of a liquid condensate film. To determine the conditions of film flow instability in an electric field at a fixed flow rate of the liquid experiments were carried out with a film running off the lining of a plane condenser set at an angle to the horizontal. The problem of eigenvalues of the Orr-Sommerfeld equation [2] is solved for the limit case of small Reynolds numbers $R \ll 1$.

1. Input equations. Let us consider the unsteady flow of a conducting liquid film on one wall of a plane channel of width l formed by two vertical electrodes. We select the system of orthogonal coordinates in which the direction of the x-axis coincides with that of the force of gravity and the y-axis is normal to the electrodes. The liquid is contained in region $-\infty \leqslant x \leqslant +\infty$, $0 \leqslant y \leqslant h(x, t)$ and the remaining space between the electrodes is filled with a nonconducting gas at rest. The electrodes are under a fixed potential difference V_0. In the electrohydrodynamic approximation the considered plane flow is defined by the system of equations [3]

$$\triangle\varphi = 0, \quad \operatorname{div} \mathbf{v} = 0, \quad \partial\mathbf{v} / \partial t + (\mathbf{v}\nabla)\,\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\nabla\mathbf{v} + \mathbf{g} \qquad (1.1)$$

with boundary conditions of the form

$$y = 0,\ v_x = 0,\quad v_y = 0;\quad y = l,\quad \varphi = V_0 \qquad (1.2)$$

$$y = h(x, t),\ \varphi = 0,\ v_y = \frac{\partial h}{\partial t} + v_x\frac{\partial h}{\partial x},\ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} + \qquad (1.3)$$

$$\frac{2h'}{1 - h'^2}\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y}\right) = 0, \quad p - p_a = -\frac{1 + h'^2}{8\pi}\left(\frac{\partial\varphi}{\partial y}\right)^2 +$$

$$\frac{2\rho\nu}{1 + h'^2}\left[\frac{\partial v_y}{\partial y} - h'\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) + h'^2\frac{\partial v_x}{\partial x}\right] - \frac{\alpha h''}{(1 + h'^2)^{3/2}}$$

where φ is the potential of the electric field, $\mathbf{v} = (v_x, v_y)$ is the velocity, g is the acceleration of gravity, ρ and p are, respectively, the density and the pressure of the liquid, p_a is the pressure of gas, ν is the kinematic viscosity coefficient, α is the surface tension corfficient, and the prime denotes derivatives with respect to x.

The problem (1.1) – (1.3) has an exact solution which determines the laminar flow of a film with a free surface. In that case the distribution of parameters is defined by

$$\varphi = E_0(h - y), \qquad E_0 = \frac{V_0}{h - l}, \qquad h = \sqrt[3]{\frac{3\nu Q}{g}} \qquad (1.4)$$

$$v_x = \frac{gh^2}{\nu}\left[\frac{y}{h} - \frac{1}{2}\left(\frac{y}{h}\right)^2\right], \quad v_y = 0, \quad p = p_a - \frac{E_0^2}{8\pi}$$

It is assumed in what follows that the flow rate Q of the liquid is independent of time. We assume the flow to be of the long-wave kind, i.e. $\varepsilon = h/\lambda \ll 1$ and $h' \lesssim \varepsilon$, where λ is a characteristic dimension of the free surface nonuniformity along the x-axis.

Estimates of the order of magnitude of quantities appearing in equations and boundary conditions show that in the flow region

$$\frac{\partial^2 v_x}{\partial x^2} \Big/ \frac{\partial^2 v_x}{\partial y^2} = O(\varepsilon^2), \quad p(x, h, t) - p(x, 0, t) = O(\varepsilon^2) + O(\varepsilon^2 R^{-1}) \quad (1.5)$$

and at the free surface

$$\frac{\partial v_x}{\partial y} = O(\varepsilon^2), \quad p - p_a + \frac{1}{8\pi}\left(\frac{\partial \varphi}{\partial y}\right)^2 + \alpha \frac{\partial^2 h}{\partial x^2} = O(\varepsilon R^{-1})$$

where R is the Reynolds number based on the mean film thickness and mean longitudinal velocity. These estimates show that for $R \gtrsim 1$ the pressure across the film is constant and is determined by surface tension forces and the distribution of the charge surface density, and that at the free surface

$$y = h(x, t), \quad \frac{\partial v_x}{\partial y} = 0, \quad p - p_a = -\frac{1}{8\pi}\left(\frac{\partial \varphi}{\partial y}\right)^2 - \alpha \frac{\partial^2 h}{\partial x^2} \quad (1.6)$$

Taking into account (1.5) and (1.6) we can write the equation of motion in the projection of the x-axis as follows:

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \frac{1}{8\pi\rho} \frac{\partial}{\partial x}\left(\frac{\partial \varphi}{\partial y}\right)^2_{y=h(x,t)} + \quad (1.7)$$

$$\frac{\alpha}{\rho}\frac{\partial^3 h}{\partial x^3} + \nu \frac{\partial^2 v_x}{\partial y^2} + g$$

To simplify the analysis of stability of the laminar flow (1.4) we use the method of averaging functions v_x and v_y proposed in [4]. We assume that approximately

$$v_x = 3u(x, t)\left\{\frac{y}{h(x, t)} - \frac{1}{2}\left[\frac{y}{h(x, t)}\right]^2\right\} \quad (1.8)$$

where $u(x, t)$ is the mean longitudinal velocity over the cross section. The profile defined by (1.8) satisfies the boundary conditions (1.2) and (1.6), and in the case of a stationary laminar flow is the same as derived by the exact solution (1.4). The equation of continuity readily yields the transverse velocity distribution

$$v_y = \frac{3}{2}\left(\frac{y}{h}\right)^2\left(u\frac{\partial h}{\partial x} - h\frac{\partial u}{\partial x}\right) + \left(\frac{y}{h}\right)^3\left(u\frac{\partial h}{\partial x} - \frac{1}{2}h\frac{\partial u}{\partial x}\right) \quad (1.9)$$

Substituting (1.8) and (1.9) into the equation of motion (1.7) and averaging over the film thickness, we obtain

$$\frac{\partial u}{\partial t} - \frac{u}{2h}\frac{\partial h}{\partial t} + \frac{9u}{10}\frac{\partial u}{\partial x} - \frac{3u^2}{10h}\frac{\partial h}{\partial x} = \quad (1.10)$$

$$\frac{1}{8\pi\rho}\frac{\partial}{\partial x}\left(\frac{\partial \varphi}{\partial y}\right)^2_{y=h(x,t)} + \frac{\alpha}{\rho}\frac{\partial^3 h}{\partial x^3} - \frac{3\nu u}{h^2} + g$$

The kinematic condition (1.3) at the free surface can be represented in the integral form

$$\partial h / \partial t + \partial uh / \partial x = 0 \quad (1.11)$$

In what follows we use as input equations (1.10) and (1.11) and the Dirichlet problem for the Laplace equations (1.1) – (1.3).

2. Convective instability. Let us assume that continuously acting infinitely small perturbations of specified frequency ω are imposed at some point x_0 on the laminar flow, so that for $x > x_0$

$$u = u_0 + u_1, \quad h = h_0 + h_1, \quad \varphi = \varphi_0 + \varphi_1 \tag{2.1}$$

where the subscripts 1 and 0 denote perturbation and the stationary solution (1.4), respectively. Let us investigate the behavior of perturbations propagating downstream. We introduce the substitution $\eta = y - h_0$. Substituting (2.1) into the equations and boundary conditions, we obtain in linear approximation

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial\eta^2} = 0, \quad \varphi(x, 0, t) = E_0 h, \quad \varphi(x, l, t) = 0 \tag{2.2}$$

$$\frac{\partial h}{\partial t} + u_0\frac{\partial h}{\partial x} + h_0\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} - \frac{u_0}{2h_0}\frac{\partial h}{\partial t} + \frac{9u_0}{10}\frac{\partial u}{\partial x} - \frac{3u_0^2}{10h_0}\frac{\partial h}{\partial x} = -\frac{E_0}{4\pi\rho}\frac{\partial^2\varphi}{\partial x\,\partial\eta}\Big|_{\eta=0} + \frac{\alpha}{\rho}\frac{\partial^3 h}{\partial x^3} - \frac{3\nu u}{h_0^2} + \frac{6\nu u_0 h}{h_0^3}$$

(here and subsequently the subscript 1 is omitted).

We seek particular solutions of system (2.2) in the form of travelling waves propagating in the positive direction of the x-axis

$$u = Ae^{i(kx-\omega t)}, \quad h = Be^{i(kx-\omega t)}, \quad \varphi = \chi(\eta)e^{i(kx-\omega t)}$$

It is readily seen that h and φ are related by $\varphi = E_0he^{-k\eta}$. The wave vector k is not known a priori and is to be determined in the course of solving the problem. We introduce the dimensionless quantities

$$c = \frac{\omega h_0}{u_0W^{1/2}}, \quad \varkappa = \frac{kh_0}{W^{1/2}}, \quad W = \frac{\rho u_0^2 h_0}{\alpha}, \quad S = \frac{E_0^2}{4\pi\rho u_0^2}$$

$$R = \frac{u_0h_0}{\nu}, \quad q = SW^{1/2}, \quad z = RW^{1/2}$$

where R is the Reynolds number, S is a parameter of electrohydrodynamic interaction, and W is the Weber number. The condition of existence of a nontrivial solution of system (2.2) yields the dispersion equation

$$z(\varkappa^4 - q\varkappa^3 - 1.2\varkappa^2 + 2.4c\varkappa - c^2) + 9i\varkappa - 3ic = 0 \tag{2.3}$$

In the general case $\varkappa(c)$ is a multiple-valued complex function which depends on parameters q and z. Only that branch of $\varkappa_1(c)$ for which $\varkappa_1(0) = 0$ has any physical meaning. The perturbations are attenuated for $\operatorname{Im}\varkappa_1 > 0$, and for $\operatorname{Im}\varkappa_1 < 0$ they increase in region $x > x_0$. If $\operatorname{Im}\varkappa_1 = 0$, we have indifferent stability. Let us investigate the behavior of $\operatorname{Im}\varkappa_1$. Separating in (2.3) the real and imaginary parts and eliminating c, we obtain

$$z\varkappa_r^2(\varkappa_r^2 - q\varkappa_r - 3) - \varkappa_i[9 + \varkappa_r^2z^2(6.4\varkappa_r^2 - 5.2q\varkappa_r + 0.96)] - \varkappa_i^2\varkappa_r^2z^3[1.78\varkappa_r^4 - 2.67q\varkappa_r^3 + \varkappa_r^2(q^2 - 0.21) + 0.32q\varkappa_r - 0.13] - \varkappa_i^2z(6\varkappa_r^2 - 3q\varkappa_r - 15.6) + \varkappa_i^3z^2(14.4\varkappa_r^2 - 6q\varkappa_r - 7.68) + \varkappa_i^4z + \varkappa_i^4z^3[3.56\varkappa_r^3(\varkappa_r - q) + \varkappa_r^2(0.67q^2 - 3.41) + 1.81q\varkappa_r + 0.77] - 1.6\varkappa_i^5z^2 + \varkappa_i^6z^3[0.64 - (0.44\varkappa_r - 1.1q)^2] = 0$$

$$\varkappa_r = \operatorname{Re}\varkappa_1, \quad \varkappa_i = \operatorname{Im}\varkappa_1$$

This equation shows that for any finite q and z there exists in the plane $\varkappa_r$, $\varkappa_i$ a curve with second-order tangency to the $\varkappa_i$-axis at the origin of coordinates, which intersects that axis at points $a_{1,2} = 0.5\,[q \pm (q^2 + 12)^{1/2}]$. We denote that curve by Γ. For $z \ll 1$ it is possible to obtain the equation for Γ in the explicit form along a certain finite interval of the $\varkappa_r$-axis, which contains the coordinate origin. Expanding the algebraic function $\varkappa_i$ by the Newton's diagram method [5] in increasing powers of z and restricting the expansion to its first terms, we obtain the following six expansions:

$$9\varkappa_{i1} = z\varkappa_r^2(\varkappa_r^2 - q\varkappa_r - 3), \quad \varkappa_{i2} = \sqrt[3]{\frac{9}{z}}, \quad \varkappa_{i3} = \sqrt[3]{\frac{9}{z}}\,\frac{i\sqrt{3}-1}{2} \qquad (2.4)$$

$$\varkappa_{i4} = -\sqrt[3]{\frac{9}{z}}\,\frac{i\sqrt{3}+1}{2}, \quad \varkappa_{i5,6} = \frac{15}{z\,(12 \mp 20\varkappa_r \pm 5q)}$$

Expansion $\varkappa_{i1}$ is the equation of Γ. Curve of this function is shown in Fig. 1. It is possible to show that condition $c = 3\varkappa_1$ is satisfied at point $(a_1, 0)$. Hence perturbations induced at frequency $\omega_* = 1.5\, u_0 h_0^{-1}\beta$, where $\beta = SW + (S^2W^2 + 12\,W)^{1/2}$, propagate downstream with a constant amplitude. Expressing the wave length of such perturbations in terms of parameters of the unperturbed flow $\lambda = 4\pi h_0 \beta^{-1}$, we find that $\beta \ll 4\pi$ is the condition of applicability of the system of equations of the long-wave approximation.

It is seen from Fig. 1 that $\varkappa_i < 0$ for $0 < \varkappa_r < a_1$ and $\varkappa_i > 0$ when $\varkappa_r > a_1$. Along curve Γ function $c\,(\varkappa_1)$ is real, and c increases with increasing distance from the coordinate origin. Since for any q and z curve Γ intersects the semiaxis $\varkappa_r > 0$ only at a single point, hence for $\omega < \omega_*$ we have $\varkappa_i < 0$. Thus the instability of the considered flow is of the convective kind with respect to perturbations of frequencies lower than ω_*. If, however, $\omega > \omega_*$, the perturbations become attenuated in region $x > x_0$. The expression for ω_* implies that the electric field has a destabilizing effect. Physically this is explained by the increase of the charge surface density in proximity to wave crests, owing to the increase of curvature of the free surface induced by the superposed perturbations. As the result, the surface forces induced by the field and acting on the wave crests tend to increase thus reducing the stabilizing effect of surface tension forces. The presence of waves at the surface of liquid leads to a much earlier breakdown of a dielectric in contact with a liquid electrode than the breakdown in the presence of plane solid electrodes [6, 7].

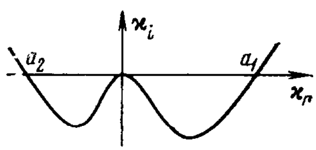

Fig. 1

3. Absolute stability. Let us suppose at the instant of time $t = 0$ perturbations

$$u\,(x, 0) = f_1(x), \quad h\,(x, 0) = f_2(x), \quad x_1 \leqslant x \leqslant x_2 \qquad (3.1)$$

appear along a finite section of the flow. We assume that $f_1(x)$, $f_2(x)$ and $\varphi\,(x, \eta, 0)$ are related by appropriate congruence conditions. Further development of perturbations is defined by system (2.2) with initial conditions (3.1). Perturbations may either infinitely increase in any region fixed with respect to x, or remain finite with increasing time t. In the first case the flow is called absolutely unstable [8], and in the second, absolutely stable. The considered flow is absolutely stable (*) (see footnote at the next page).

To prove this statement we express each of the quantities u and h in terms of Fourier integrals in k of the form

$$\Lambda(x,t)=\sum_{j=1}^{2}\int_{-\infty}^{\infty}\psi(k)\,e^{i[kx-\omega_j(k)t]}dk \qquad (3.2)$$

It is possible to express $\varphi(x,\eta,t)$ by a similar formula with the integrand containing the factor $e^{-k\eta}$. Functions $\psi(k)$ are determined by initial conditions (3.1), and $\omega_j(k)$ are single-valued branches of function $\omega(k)$ which is defined by the dispersion equation (2.3). The dimensionless form of this function is

$$c=1.2\varkappa-1.5iz^{-1}+\sqrt{P(\varkappa)}$$
$$P(\varkappa)=\varkappa^4-q\varkappa^3+0.24\varkappa^2+5.4iz^{-1}\varkappa-2.25z^{-2} \qquad (3.3)$$

We distinguish branches $\omega_1(k)$ and $\omega_2(k)$ by their value at the coordinate origin, viz. $\omega_1(0)=0$ and $\omega_2(0)=-3\nu h_0^{-2}i$. Let us investigate the behavior of $\operatorname{Im}\omega_1$. Assuming c to be complex and $\varkappa$ real, we separate in (2.3) the real and imaginary parts and eliminate $\operatorname{Re} c$. We obtain

$$z\varkappa^2(\varkappa^2-q\varkappa-3)+c_i[3+1.33z^2\varkappa^3(\varkappa-q)+0.32z^2\varkappa^2]+c_i^2[5z+$$
$$0.44z^3\varkappa^3(\varkappa-q)+0.11z^3\varkappa^2]+2.67z^2c_i^3+0.44z^3c_i^4=0,\quad (c_i=\operatorname{Im}c)$$

It can be seen that for any q and z there is in the plane $\varkappa$, c_i a curve tangent to the $\varkappa$-axis at the coordinate origin which intersects that axis at points a_1 and a_2. For small z that curve can be defined by the equation $3c_i=z\varkappa^2(3+q\varkappa-\varkappa^2)$. This implies that $\operatorname{Im}\omega_1(\sigma_1)=0$ and $\operatorname{Im}\omega_1(\sigma_2)=0$, where $\sigma_{1,2}=0.5h_0^{-1}\,[SW\pm(S^2W^2+12W)^{1/2}]$. Along the infinite intervals $-\infty<k<\sigma_2$ and $\sigma_1<k<+\infty$ we have $\operatorname{Im}\omega_1<0$, so that for $t\to\infty$ and $\varepsilon>0$

$$\int_{-\infty}^{\sigma_2-\varepsilon}\psi(k)\,e^{i[kx-\omega_1(k)t]}\,dk\to 0,\qquad \int_{\sigma_1+\varepsilon}^{+\infty}\psi(k)\,e^{i[kx-\omega_1(k)t]}\,dk\to 0$$

There are also two finite intervals $\sigma_2<k<0$ and $0<k<\sigma_1$ along which $\operatorname{Im}\omega_1>0$. The monochromatic components of spectral expansions (3.2) which correspond to these intervals increase with $t\to\infty$. For real k the branch $\omega_2(k)$ lies in the lower half-plane, hence the second term in (3.2) tends to zero for $t\to\infty$.

Let us prove that in the first term of (3.2) the integration path between points $k_2=\sigma_2-\varepsilon$ and $k_1=\sigma_1+\varepsilon$ with $\varepsilon>0$ of the real axis can be shifted so as to have the condition $\operatorname{Im}\omega_1(k)<0$ satisfied all along the new path. It can be readily shown that for perturbations which are localized at the initial instant of time $\psi(k)$ is an entire function in the complex plane k. Owing to this the integrand function in (3.2) has no singularities, except the points of branching of function $\omega(k)$. The behavior of Λ, thus, depends on the analytic properties of function $c(\varkappa)$.

Let us first, consider the case of $z\ll 1$. In the complex plane $\varkappa$ (Fig. 1) curve Γ is constructed so that along it $\operatorname{Im} c_1=0$. Since along the intervals $-\infty<\varkappa<a_2$ and $a_1<\varkappa<+\infty$ of the real axis $\operatorname{Im} c_1<0$, hence, owing to the continuity of function $c_1(\varkappa)$ we find that $\operatorname{Im} c_1<0$ lies in the region below curve Γ. Branching points of function $c(\varkappa)$ can only be found among the zeros of polynomial $P(\varkappa)$. Let us determine these. Expanding function $\varkappa$ which is defined by the equation $P(\varkappa)=0$,

*) The absolute stability for $S=0$ was proved in [9]. The method used in [9] is applicable to the considered problem only for $z\ll 1$.

in increasing powers of z and restricting the expansion to the first two terms, we obtain

$$\varkappa_{1,2} = \pm\sqrt{\frac{3}{2z}} + \frac{1}{4}q - \frac{9}{10}i, \qquad (3.4)$$

$$\varkappa_{3,4} = i\left(\pm\sqrt{\frac{3}{2z}} + \frac{9}{10}\right) + \frac{1}{4}q$$

Comparing the first formula of (2.4) and (3.4), we find that for $z \ll 1$ the branching points of function $c(\varkappa)$ cannot lie in region G bounded by the part of curve Γ in the lower half-plane and the segment a_2a_1 of the real axis of the complex plane $\varkappa$. Let us show that for any $0 < z < +\infty$ the branching points cannot lie on the boundary of region G. It is readily seen that the equation $P(\varkappa) = 0$ has no real roots, hence branching points cannot lie on segment a_2a_1. They can neither lie on the remaining part of the boundary of region G, since otherwise we would have at Γ $\operatorname{Im} c_1 \neq 0$. Since function $c(\varkappa)$ is continuous with respect to parameter z, $z \neq 0$, hence for any z branching points cannot lie in region G.

It is, thus possible to separate in some finite simply connected region wholly containing G a single-valued branch $c_1(\varkappa)$. With the use of the Cauchy integral theorem the integration path for the first term in (3.2) can be shifted so that all along the new path the condition $\operatorname{Im} \omega_1 < 0$ is satisfied. In virtue of this $\Lambda \to 0$ for $t \to \infty$.

Since in reality the region occupied by the film is bounded, hence for a fairly small initial amplitude perturbations leave that region before separation of the laminar flow takes place. Thus for $SW + (S^2W^2 + 12\,W)^{1/2} \ll 4\pi$ and $R \gtrsim 1$ the considered flow is absolutely stable.

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